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A MODEL TO ESTIMATE THE VIRGIN AND ULTIMATE EFFECTIVE STIFFNESSES FROM THE RESPONSE OF A DAMAGED STRUCTURE TO A SINGLE EARTHQUAKE

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SUMMARY

The paper presents a model describing the evolution of the effective stiffness of a 'mode' of a structure subjected to an earthquake engaging it in its non-linear phase of behaviour. Such models are available in the literature; however, the proposed one allows the estimation of the effective stiffnesses both of the virgin state of the structure and of its ultimate state, even when the excitation considered does not act on an undamaged system nor leads it to ultimate conditions. Only the base input and the response at a significant location of the structure need to be known to apply the procedure. This is satisfactorily checked against the results of shaking table tests performed on masonry buildings.

KEY WORDS: effective stiffness; identification; damage indices

1. INTRODUCTION

1.1. General

The development of procedures for system identification from earthquake records enables reliable estimates of modal and structural properties of buildings for which both base input and responses at one or more locations are available. At the occurrence of non-linear response, appropriate procedures are employed and quite often linear methods are used to supply parameters that are equivalent, during the considered motion, to the non-linear actual response.

However, the results of such identifications cannot be used to assess the dynamic response to a base excitation whose 'severity' is significantly different from that for which identification was carried out, since modal characteristics exhibited by the structure during the two events may differ considerably. Similarly, modal parameters related to the actual (damaged) configuration cannot be compared to those pertaining to the virgin (undamaged) configuration, when the original design specifications are missing, as often is the case. Thus, two questions regarding the evaluation of the seismic safety of damaged structures cannot reliably be answered. They are: (1) how far are the present structural conditions from those of the original undamaged state? (2) how far are the present structural conditions from the ultimate ones?

This work attempts to answer the two questions through the analysis of the evolution, during the response to an earthquake inducing non-linear behaviour, of the effective stiffness of a dominant 'mode' with respect to relative displacements at a significant point of the structure. More details on the concepts and definitions used in the following are now presented.

1.2. Background

Consider the equation describing the response of a multi d.o.f. system to a ground motion:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) = -\mathbf{M}\mathbf{I}a_g \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\ddot{\mathbf{x}}$ the relative acceleration vector, $\mathbf{f}$ the restoring forces vector, a_g the base acceleration and $\mathbf{I}$ the identity matrix. Restoring forces account for the type of the structural response: in the

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linear case (with the usual meanings of **k** and **c**): $\mathbf{f} = \mathbf{c}\dot{\mathbf{x}} + \mathbf{k}\mathbf{x}$, whereas in the case of non-linear behavior **f** accounts for the physical effects occurring in the system during the response (e.g. viscous and hysteretic phenomena) and is generally unknown in its form and parameters. Examples of identifying these items can be found in the works of Iwan,[1] Peng[2] and Cifuentes.[3]

To get information about **f**, a 'modal' decomposition may be attempted; although modes do not strictly exist for a non-linear system, filtering allows to separate the response into 'mode-like' contributions, dominated by a narrow-frequency content[2]. After having isolated the generic rth 'mode' contribution to the absolute response acceleration and making reference to the ith location, equation (1) reads

$$h_i^r(x_i^r, \dot{x}_i^r) = -\beta_i^r a_g - \ddot{x}_i^r \tag{2}$$

where h_i^r is the modal restoring force per unit mass, β_i^r the effective participation factor at the considered location and $\dot{x}_i^r$, x_i^r the contributions of the selected 'mode' to the relative velocity and displacement. Note that the resulting signal h_i^r is unable to distinguish between the two components of the r.h.s. of equation (2), where β_i^r and $\ddot{x}_i^r$ are unknown. However, filtering provides a very good approximation of the 'modal' relative acceleration and, through integration, of the relative 'modal' displacements. These may be correlated to h_i^r, thus obtaining graphs like the one shown in Figure 1(a) from which information about the nature of the system may be retrived. In the figure, the widening of cycles (referred to a narrow-frequency content) shows the hysteretic nature of the structural response. Moreover, the progressive reduction of the slope of cycles accounts for the degradation of stiffness. To make the latter effect clearer, the modal effective stiffness k_{eff}^r is introduced, defined as the ratio between the value of the modal restoring force per unit mass at the maximum modal displacement during a cycle of the response at a given location to such maximum displacement $x_{i,\max,\mathrm{cycle}}^r$:

$$k_{\mathrm{eff}}^r = \left.\frac{h_i^r}{x_i^r}\right|_{x_i^r,\max,\mathrm{cycle}}$$

Figure 1(b) shows the variation of k_{eff} referred to the case represented in Figure 1(a). Both figures refer to signals recorded during one of the shaking table tests on masonry buildings carried out within a research project funded by the European Community. Tests were carried out by ISMES (Bergamo, Italy) and NTUA

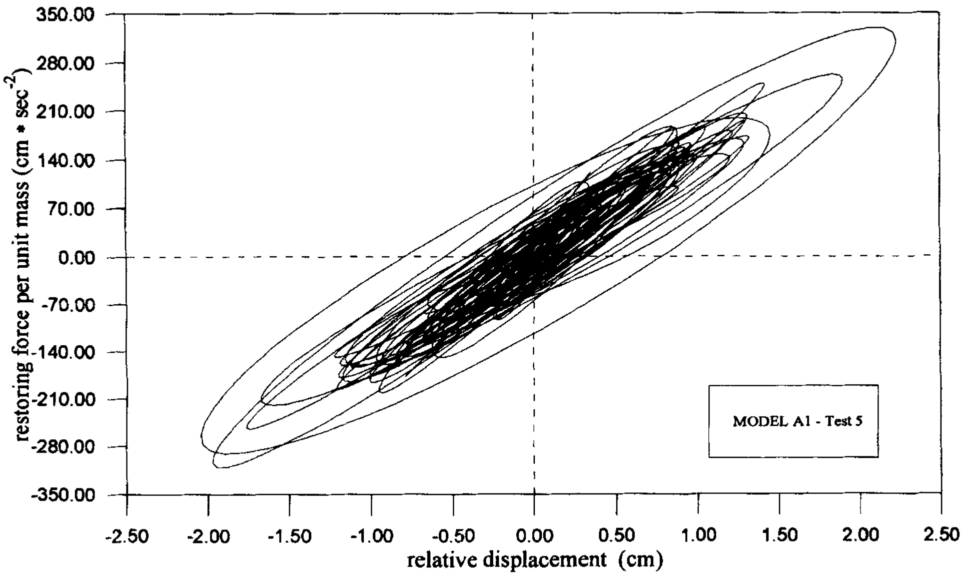


Figure 1(a). Restoring forces per unit mass vs. relative modal displacements, system A1, test 5

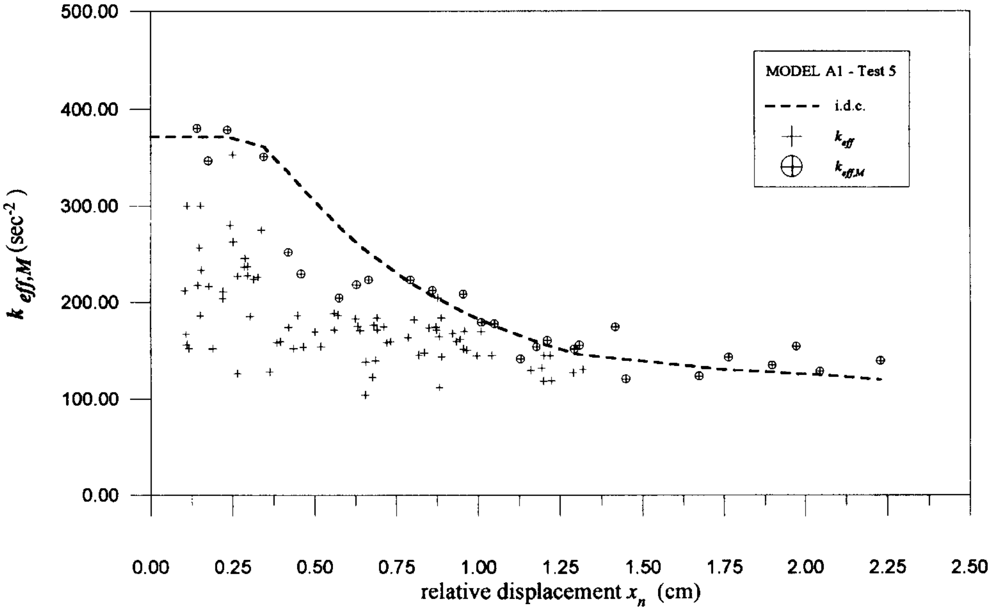


Figure 1(b). Effective stiffness of the first x-mode vs. relative modal displacements, system A1, test 5

(Athens, Greece) facilities. The program involved tests on 30 models, each subjected to a sequence of base inputs (from 4 to 7), of similar frequency content and progressively increasing peak accelerations, up to ultimate conditions. In this paper the description of the experimental program and results are not given; they can be found in Reference 4.

1.3. *Outline of the method*

As to Figure 1(b) (referred to model A1 and to test 5, whose characteristics are reported in Table I), it may be seen that the distribution of experimental points is rather erratic, especially at very low values of displacements. Moreover, a given displacement $\bar{x}_n$ corresponds to several values of k_{eff}: the maximum of these values $k_{\mathrm{eff,M}}$ refers to the cycle during which $\bar{x}_n$ is first attained. Lower values of k_{eff} at $\bar{x}_n$ refer to subsequent cycles occurring after the system suffered further degradation. The points corresponding to such maximum values $k_{\mathrm{eff,M}}$ at each $\bar{x}_n$ are marked by a circle in the figure. These points define a curve, dotted in the figure, which is the envelope of the whole set of (k_{eff}, x_n) points for the given excitation. The decreasing portion of the curve describes the degradation of stiffness at the increase of displacements.

In the following, reference will be made only to points lying on such enveloping curve. This curve will be referred to as 'degrading curve'. If it is referred to a single earthquake acting on the structure (which may be already damaged) without leading it to ultimate, as is the case of Figure 1(b), it will be denoted as 'intermediate degrading curve' (i.d.c.). A similar curve, referred to the whole set of events acting on the structure from its virgin configuration up to ultimate, will be denoted as 'global degrading curve' (g.d.c.).

The evolution of $k_{\mathrm{eff,M}}$ against modal displacements has been worked out for all the events pertaining to each model. For all models it was found that the decreasing portions of the i.d.c's, determined by taking into account separately all the earthquakes acting on the considered system, define a single curve, characterizing the decrease of the effective stiffness at the increase of relative displacements. This curve is the 'global degrading curve' for the given model. As an example, consider Figure 2, referred to system C1 and to 4 shocks of increasing peak acceleration, whose values are reported in Table I. In Figure 2 the points ($k_{\mathrm{eff,M}}$, x_n) corresponding to the four shocks are denoted by different symbols.

The similar ways in which $k_{\mathrm{eff,M}}$ decreases within a given interval Δx_n of displacements, when the 'global' or the 'intermediate' degrading curve is considered, suggest the possibility of estimating the former evolution of $k_{\mathrm{eff,M}}$ by the latter one.

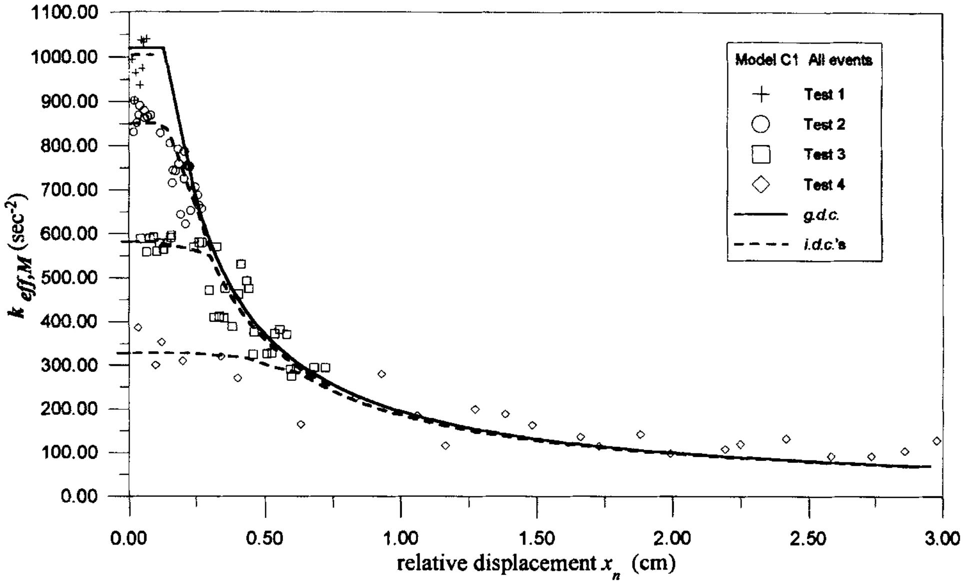


Figure 2. Global degrading curve (4 tests) of building C1. Intermediate degrading curves are shown (dotted)

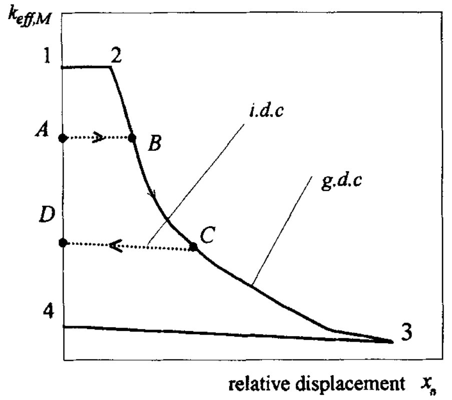


Figure 3. Qualitative global and intermediate degrading curves

Consider a structure whose g.d.c. (not known) is represented qualitatively by the curve 1-2-3-4 of Figure 3 and consider an event acting on the system when, due to previous damaging shocks, the initial stiffness is k_A. The analysis of recorded signals provides the curve A-B-C-D whose decreasing portion BC tends to overlap with the decreasing portion of the g.d.c., as said.

This paper presents a method to estimate the g.d.c. (1-2-3-4) based on the information supplied by the available i.d.c (A-B-C-D). This is made by identifying the properties of a model (derived from References 1 and 3) under the hypotheses specified in the following sections. The model is constituted by brittle elements, elastic up to failure, and by indefinitely elasto-plastic elements.

The estimate of the g.d.c. supplied by the model provides tentative values of both the original and the final $k_{eff,M}$ (k_1 and k_4). The comparison of such values to the $k_{eff,M}$ evaluated at the end of the considered earthquake (k_D) allows to attempt an answer to the two questions above and enables a more aware evaluation of the seismic safety of the damaged structure.

The comparison is carried out by means of two damage indices, evaluated in Section 3, i.e. k_D/k_1 and $(k_1 - k_D)/(k_1 - k_4)$. The first index describes how far the structure is from the original configuration after the considered shock and the second index describes to what extent the original stiffness resources have been exploited after the considered event.

The ability of the proposed model both in describing the 'intermediate' and the 'global' evolutions of $k_{eff,M}$ is checked by using experimental data of the above-mentioned experimental research. Results of such comparisons are always quite satisfactory, as will be seen.

2. THE MODEL

The model is intended to supply the analytical expression of $k_{eff,M}$ as a function of the maximum cycle displacement x_n. It is based on the assumption that the decreasing portion of the i.d.c., determined during an event engaging the structure non-linearly, is a part of the g.d.c. of the system, within the same interval of x_n. The model is derived from the one presented in Reference 3 but differs from it in the items that will be specified in the following.

The model proposed herein is constituted by brittle-elastic (BE) elements and elastoplastic (EP) elements working in parallel (Figure 4). The EP elements never break, differently from what is assumed in the model of Reference 1, where EP elements are active up to a given modal displacement. BE elements, on the other hand, cease to be active when their limiting elastic displacement is exceeded. Hence, during the (intermediate) event considered, the number of the EP elements does not change, while that of the BE elements decreases as the system suffers permanent degradation. This is described by the progressively increasing number of the 'broken' BE elements. The same features hold as far as the whole 'life' of the structure is considered.

Let N_p the number of the EP elements of the model and N_b^o the number of the BE elements existing at the origin of the 'life' of the system. Of these elements only N_b are still active at the beginning of the event for which recorded responses are available. The original total number of elements $N = N_b^o + N_p$ is arbitrarily fixed, but the original subdivision between BE and EP elements (N_b^o and N_p) is unknown and is determined during the identification.

The number of brittle elements still active at the beginning of the event considered N_b cannot be fixed arbitrarily as is made in Reference 1: here N_b has to be determined based on the response to the given earthquake. The difference $N_b^o - N_b$ (representing the number of brittle elements failed during previous unknown shocks) describes the permanent damage suffered before the considered event.

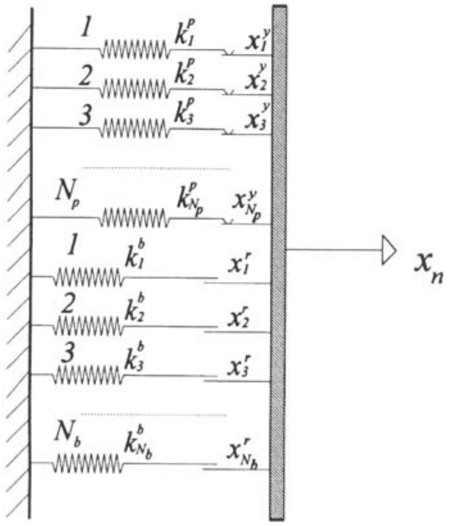


Figure 4. The model

After determining the properties of the whole set of elements and the numbers N_p, N_b^o and N_b based on the known response, the model provides both the description of the variation of $k_{eff,M}$ with x_n during the excitation considered and the estimate of the g.d.c.

The total force per unit mass developed by all the active elements at a given x_n is

$$f_n(x_n) = \sum_{i=1}^{N_p} p_i(x_n) + \sum_{j=1}^{N_b(x_n)} b_j(x_n) \tag{3}$$

where p_i and b_i are the forces developed, respectively, by the ith EP element and by the jth BE element.

The total force f_n depends on the state of each element at the considered x_n. At x_n let $e(x_n)$ be the number of EP elements still in their elastic phase, $s(x_n)$ the number of EP elements that have already yielded in a previous cycle and $r(x_n)$ the number of BE elements still active. For the generic jth BE element, let x_j^r and b_j^* be, respectively, the displacement and the force per unit mass at failure and k_j^b its stiffness; for the generic ith EP element let $x_i^y$and p_i^* be, respectively, the yielding displacement and the maximum force per unit mass and k_i^p the initial elastic stiffness.

Equation (3) modifies to

$$f_n(x_n) = \sum_{i=1}^{e(x_n)} k_i^p x_n + \sum_{i=1}^{s(x_n)} p_i^* + \sum_{j=1}^{r(x_n)} k_j^b x_n \tag{4}$$

Therefore, the effective stiffness at x_n is given by

$$k_{eff,M}(x_n) = \frac{\sum_{i=1}^{e(x_n)} k_i^p x_n + \sum_{i=1}^{s(x_n)} p_i^* + \sum_{j=1}^{r(x_n)} k_j^b x_n}{x_n} = \sum_{i=1}^{e(x_n)} k_i^p + \sum_{i=1}^{s(x_n)} \frac{p_i^*}{x_n} + \sum_{j=1}^{r(x_n)} k_j^b, \tag{5}$$

Both in equations (4) and (5), the numbers $e(x_n)$, $s(x_n)$ and $r(x_n)$ are functions of the current value of x_n. In the model these functions are expressed through the following hypotheses about the properties of BE and EP elements:

(1) equal values of maximum force per unit mass for all the elements:

$$p_i^* = b_j^* = f^* \tag{6}$$

(2) initial stiffnesses of the EP elements varying linearly with the element index:

$$k_i^p = i dk \quad dk \text{ being a constant increment} \tag{7}$$

(3) the same type of variation for stiffnesses of BE elements:

$$k_j^b = j dk \tag{8}$$

From the above hypotheses the following expressions are obtained for displacements at yield and at failure:

$$x_i^y = \frac{f^*}{i\,dk}, \qquad x_j^r = \frac{f^*}{j\,dk} \tag{9}$$

The lowest and the highest yield and failure displacements are given by

$$x_{min}^y = \frac{f^*}{N_p dk}, \quad x_{min}^r = \frac{f^*}{N_b dk}, \quad x_{max}^y = x_{max}^r = \frac{f^*}{dk} \tag{10}$$

By recalling equations (6)–(10) it can be easily seen that the following possibilities hold for $e(x_n)$ and $r(x_n)$:

$$e(x_n) = \begin{cases} N_p & \text{if } x_n \leqslant x^y_{min} \\ \dfrac{f^*}{x_n \, dk} & \text{if } x^y_{min} \leqslant x_n \leqslant x^y_{max} \\ 0 & \text{if } x_n \geqslant x^y_{max} \end{cases}$$

$$r(x_n) = \begin{cases} N_b & \text{if } x_n \leqslant x^r_{min} \\ \dfrac{f^*}{x_n \, dk} & \text{if } x^r_{min} \leqslant x_n \leqslant x^r_{max} \\ 0 & \text{if } x_n \geqslant x^r_{max} \end{cases} \tag{11}$$

By the above expressions of $e(x_n)$ and $r(x_n)$, noting that $s(x_n) = N_p - e(x_n)$, equation (5) modifies to

$$k_{eff,M}(x_n) = \sum_{i=1}^{e(x_n)} i \, dk + \sum_{i=e(x_n)+1}^{N_p} \frac{f^*}{x_n} + \sum_{j=1}^{r(x_n)} j \, dk \tag{12}$$

Carrying out the summations of equations (12), thanks to the hypotheses (6)–(8) that exist in closed form, and taking into account equation (11), the following analytical expression for $k_{eff,M}$ is found:

and

$$\begin{aligned} k_{eff,M}(x_n) = & \frac{f^{*2}}{2x_n^2 dk} [(H^r_{min} - H^y_{min})(1 - H_{max})] \\ & + \frac{f^*}{x_n} [N_p(H^y_{min} - H^y_{min} H_{max}) + H_{max} + \tfrac{1}{2}(H^r_{min} + H^y_{min})(1 - H_{max})] \\ & + \frac{N_p(N_p + 1)}{2} dk \, (1 - H^y_{min}) + \frac{N_b(N_b + 1)}{2} dk \, (1 - H^r_{min}) \end{aligned} \tag{13}$$

with

$$H(x_n - x^y_{min}) = H^y_{min}$$

$$H(x_n - x^r_{min}) = H^r_{min}$$

$$H(x_n - x^r_{max}) = H(x_n - x^y_{max}) = H_{max}$$

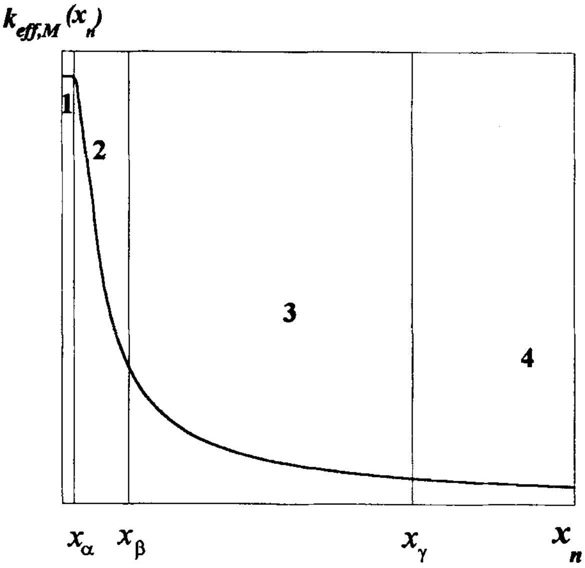
where $H(a)$ is the Heaviside unit step function given by

$$H(a) = \begin{cases} 1 & \text{for } a \geqslant 0 \\ 0 & \text{for } a < 0. \end{cases}$$

Equation (13) leads to a $(k_{eff,M}, x_n)$ curve of the type qualitatively represented in Figure 5 whose analytical expressions in the various intervals of x_n are given by:

$$k_{eff,M}(x_n) = \left[\frac{N_p(N_p + 1)}{2} + \frac{N_b(N_b + 1)}{2}\right] dk, \quad 0 \leqslant x_n \leqslant \min\left(\frac{f^*}{N_p dk}; \frac{f^*}{N_b dk}\right) \tag{14a}$$

$$k_{eff,M}(x_n) = \begin{cases} \dfrac{N_p(N_p + 1)}{2} dk + \dfrac{f^{*2}}{2x_n^2 \, dk} + \dfrac{f^*}{2x_n} & \text{if } N_p \leqslant N_b, \quad (14b) \\ -\dfrac{f^{*2}}{2x_n^2 \, dk} + \dfrac{f^*}{x_n}\left(N_p + \dfrac{1}{2}\right) + \dfrac{N_b(N_b + 1)}{2} dk & \text{if } N_p \geqslant N_b \quad (14c) \end{cases}$$



$$x_\alpha = \min\left\{\frac{f^*}{N_p dk}; \frac{f^*}{N_b dk}\right\} \qquad x_\beta = \max\left\{\frac{f^*}{N_p dk}; \frac{f^*}{N_b dk}\right\} \qquad x_\gamma = \frac{f^*}{dk}$$

Figure 5. Portions of the global $k_{\mathrm{eff,M}}$, x_n curve with different analytical expressions

$$\min\left(\frac{f^*}{N_\mathrm{p}\,\mathrm{d}k}; \frac{f^*}{N_\mathrm{b}\,\mathrm{d}k}\right) \leqslant x_n \leqslant \max\left(\frac{f^*}{N_\mathrm{p}\,\mathrm{d}k}; \frac{f^*}{N_\mathrm{b}\mathrm{d}k}\right)$$

$$k_{\mathrm{eff,M}}(x_n) = \frac{f^*}{x_n}(N_\mathrm{p} + 1), \quad \max\left(\frac{f^*}{N_\mathrm{p}\mathrm{d}k}; \frac{f^*}{N_\mathrm{b}\,\mathrm{d}k}\right) \leqslant x_n \leqslant \frac{f^*}{\mathrm{d}k} \tag{14d}$$

$$k_{\mathrm{eff,M}}(x_n) = N_\mathrm{p}\frac{f^*}{x_n}, \quad \frac{f^*}{\mathrm{d}k} \leqslant x_n \leqslant \infty \tag{14e}$$

Four parameters have hence to be identified in order to define the model, i.e. N_p (or N_b^o with $N = N_\mathrm{b}^\mathrm{o} + N_\mathrm{p}$), N_b, f^*, and $\mathrm{d}k$.

Identification is carried out by minimizing, with respect to the four above-mentioned parameters, the following error function, where N is fixed:

$$\varepsilon(\boldsymbol{\alpha}) = \sqrt{\sum_{n=1}^{c} [\hat{k}_{\mathrm{eff,M}}(x_n, \boldsymbol{\alpha}) - k_{\mathrm{eff,M}}(x_n)]^2} \tag{15}$$

where $\boldsymbol{\alpha} = (f^*, N_\mathrm{b}, N_\mathrm{p}, \mathrm{d}k)$ is the vector of the quantities to be identified, $k_{\mathrm{eff,M}}(x_n)$ is the maximum value of the experimental effective stiffness at x_n, c is the number of the experimental points of the i.d.c. supplied by the responses recorded during the considered earthquake and $\hat{k}_{\mathrm{eff,M}}(x_n, \boldsymbol{\alpha})$ is the effective stiffness evaluated through equation (13) given the current set of parameters $\boldsymbol{\alpha}$.

After identifying the parameters of the model based on the experimental i.d.c., the expression of the g.d.c. can be obtained from equation (14) by putting $N_\mathrm{b} = N_\mathrm{b}^\mathrm{o}$. The proof of the uniqueness of the solution is complicated by the fact that $k_{\mathrm{eff,M}}$ has different analytical expressions, depending on the section of the curve to which it refers. A detailed treatment of this item appears in Reference 5. The outline of the proof is as

follows. Suppose that there exist two different sets of the model parameters, $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$, for which the error is zero:

$$e(\boldsymbol{\alpha}_1) = \varepsilon(\boldsymbol{\alpha}_2) = 0 \tag{16}$$

It follows that $\hat{k}_{\mathrm{eff,M}}(x_n, \boldsymbol{\alpha}_1) = \hat{k}_{\mathrm{eff,M}}(x_n, \boldsymbol{\alpha}_2)$ for all the c values of x_n. This condition gives rise to a set of linear and non-linear algebraic equations in which the parameters $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ appear. The total number of equations E depends on which part of the curve of Figure 5 contains the maximum of the c displacements x_n determined by the earthquake considered. In case it falls within the part 2 of the curve, $E = 4$; if it falls within the part 3, $E = 7$; and if it falls within part 4; $E = 10$. By using assumptions that are fully consistent with the physical meaning of the quantities to be identified ($\mathrm{d}k > 0, f^* > 0$, $0 \leqslant N_\mathrm{b} \leqslant N, 0 \leqslant N_\mathrm{b}^\mathrm{o} \leqslant N$) it can be proved that $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_2$.

However, uniqueness exists only if the response to the earthquake considered engages, even very slightly, the non-linear behaviour of the system, i.e. if the 'experimental' points ($k_{\mathrm{eff,M}}$, x_n) do not fall only in the part 1 of the curve of Figure 5.

It may be noted that the presentation of the model has been limited to the description of the functions correlating $k_{\mathrm{eff,M}}$ to x_n. The model allows also the description of the r.f. at a generic displacement x, for which $|x| \leqslant |x_n|$, thus enabling to follow the complete evolution of k_{eff} (and not only of $k_{\mathrm{eff,M}}$) during the considered response. This presentation is made in Reference 5.

3. APPLICATIONS

The proposed method is checked against the results provided by shaking table tests carried out within the experimental research program described in Reference 4. Due to space limitations, only two models A1 and C1, to which reference was made previously, will be considered. Both models are brick masonary buildings with wooden slabs. However, in model A1 the connection of slabs to the supporting walls is improved by steel ties, while in C1 these devices are lacking. Figure 6 shows the two models while being repaired. Base excitation is three-directional, along axes x, y, z, z being the vertical axis and x the horizontal axis along the longitudinal direction (walls with doors and windows). Responses in x-direction at the midpoint of the second storey are considered to work out $k_{\mathrm{eff,M}}$, with major damage been suffered by x-walls in both cases. Results of shaking table tests and those provided by the proposed procedure are referred to the real scale. Table I shows the peak values of base accelerations in x and y directions of the sequel of shocks acting on the two considered models.

Due to the non-existence of a unique solution in the case of linear response, only the last 3 excitations are considered in both cases. In fact, the analysis of recorded data shows that during the first two base inputs the system A1 behaves linearly and the same happens for system C1 during the first input set.

The comparison of experimental results to those obtained from the proposed method is carried out according to the following scheme. First, signals pertaining to all the base inputs of Table I are processed to obtain the g.d.c. curves for the two systems and for the first x-mode. These provide the 'true' values of $k_{\mathrm{eff,M}}$ at the beginning and at the end of the 'life' of the two structures. Then the model is separately applied to the events engaging the systems non-linearly (inputs 3, 4, 5, for A1 and inputs 2, 3, 4 for C1). After identifying the relevant parameters, the three calculated g.d.c.'s are finally determined for each building. These allow to estimate the initial and final values for $k_{\mathrm{eff,M}}$, which are compared to the ones experimentally obtained in step 1.

The comparison is carried out by defining the following error indices, which are functions of the initial and final effective stiffness supplied by the experimental (one curve per model) and calculated (three curves per model) g.d.c.'s:

$$e_1 = \frac{(k_1^\mathrm{r} - k_1^\mathrm{s})}{k_1^\mathrm{r}}, \quad e_4 = \frac{(k_4^\mathrm{r} - k_4^\mathrm{s})}{k_4^\mathrm{r}}$$

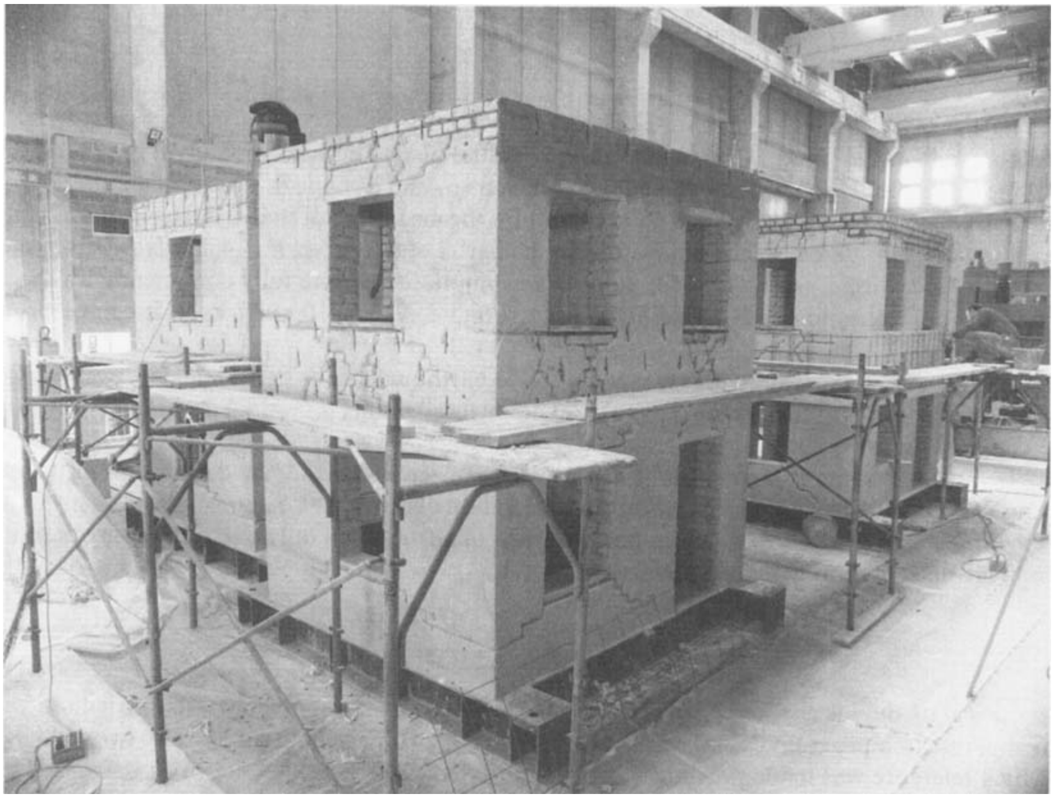


Figure 6. Some of the tested models under repair

Table I. Peak accelerations of base inputs. Buildings A1 and C1

A1			C1		
Test	a_x (m/s^2)	a_y (m/s^2)	Test	a_x (m/s^2)	a_y (m/s^2)
1	0·54	0·63	1	0·60	0·70
2	0·96	0·96	2	1·75	1·82
3	1·76	1·52	3	2·57	2·55
4	2·46	2·13	4	2·74	2·90
5	3·17	2·70			

where suffix r refers to experimentally determined quantities and suffix s to the ones provided by the proposed method.

Further, the two damage indices defined in Section 1, i.e.

$$d_1^{\mathrm{r,s}} = \frac{k_{\mathrm{D}}^{\mathrm{r}}}{k_1^{\mathrm{r,s}}}, \quad d_4^{\mathrm{r,s}} = \frac{(k_1^{\mathrm{r,s}} - k_{\mathrm{D}}^{\mathrm{r}})}{(k_1^{\mathrm{r,s}} - k_4^{\mathrm{r,s}})}$$

are calculated with respect to the stiffness values obtained both by the experimental g.d.c. (suffix r) and by the calculated ones (suffix s). The comparison between the experimentally derived and model-supplied damage

indices provides a further assessment of the capacity of the model to predict the g.d.c. of the system by only knowing one i.d.c.

The above-mentioned error and damage indices are computed by using the initial (k_1) and the final (k_4) values of $k_{\mathrm{eff,M}}$. The model gives directly, by putting in equation (14a) $N_b = N_b^o$, the original stiffness value k_1 estimated based on the considered intermediate event. However, the determination of the experimental k_1 is affected by the irregular distribution of the experimental points occurring at very low displacements, as can be seen from Figures 7 and 8. In the following examples, k_1 is determined by averaging the values of $k_{\mathrm{eff,M}}$ evaluated for the tests during which the systems behaved linearly (tests 1 and 2 for A1 and test 1 for C1).

For a given tested system the final values $k_4^{r,s}$ are determined by getting from the experimental g.d.c. and from the calculated ones the values corresponding to the experimental maximum displacement achieved during the whole sequence of tests on the given building. This occurred in the shock during which the system suffered heavy damage but no collapses. Hence, no ambiguity exists as far as the evaluation of the above indices for the tested buildings is concerned.

However, the application of the present method to real cases requires a criterion for the evaluation of the final stiffness k_4 based on the calculated g.d.c. In fact, in such cases the only available information refers to the response to the considered earthquake, which possibly increased the existing state of damage (and the consequent non-linear characteristics of the structure), without reaching ultimate conditions. If this were the case, the evaluations proposed in this work could still be carried out, but would have little sense, the building being practically at the end of its life. So displacements characterizing the ultimate conditions will not be available in practice. On the other hand, the word 'ultimate' is somehow conventional, depending on the physical conditions of the building that are considered compatible with its survival and on what is considered as 'heavy damage'. For instance, during the above-mentioned experimental program, the sequence of shocks was interrupted when damage was visually so severe that it was feared that another test would have produced the total collapse, destroying measurement devices and preventing the structure to be moved off the table and eventually repaired. Structural conditions in such instances were conventionally assumed as the ultimate ones, although the real ultimate state might have been slightly different.

The criterion that is suggested for the derivation of damage indices in real situations is to estimate k_4^s at displacements for which the derivative of the calculated g.d.c. is very small. In this way one cannot be sure

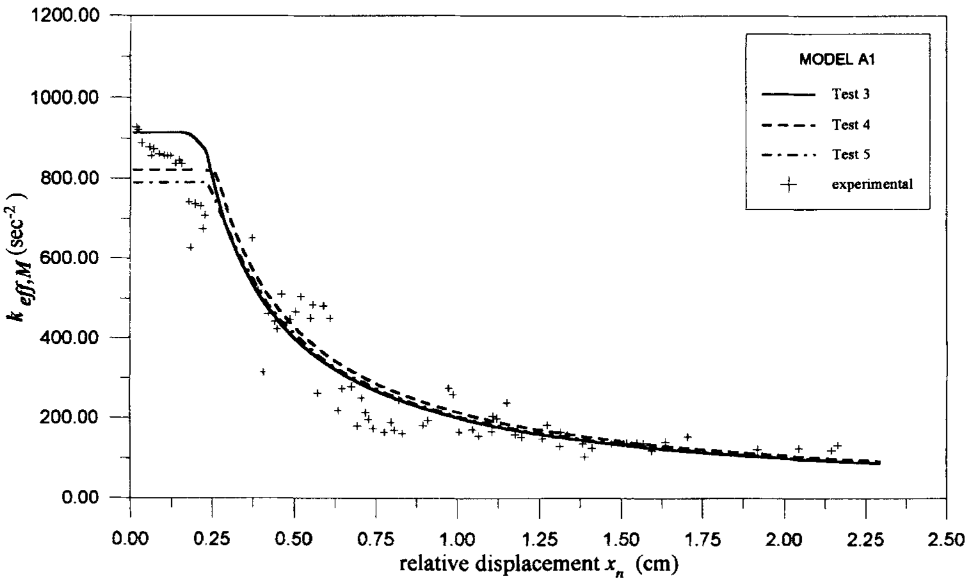


Figure 7. Experimental and calculated g.d.c.'s: system A1

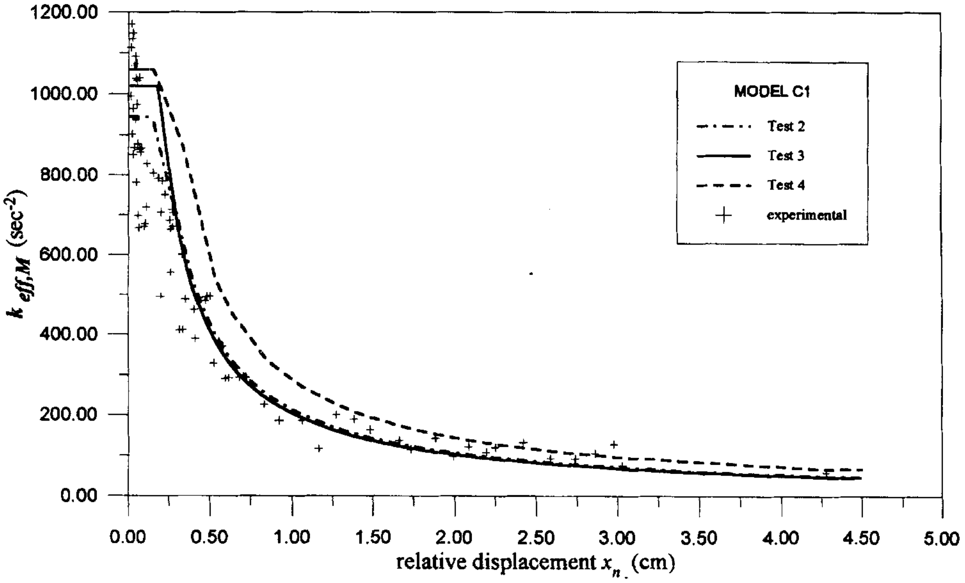


Figure 8. Experimental and calculated g.d.c.'s: system C1

Table II. Identified model quantities

A1					C1				
Test	f^*	dk	N_b	N_b^o	Test	f^*	dk	N_b	N_b^o
3	3·40	0·35	29	42	2	3·0	0·32	18	30
4	3·90	0·32	20	46	3	4·0	0·40	10	50
5	4·20	0·33	5	52	4	6·0	0·40	1	60

that the estimated value is the real one, though the latter will not differ significantly from it. But this uncertainty is poorly reflected in the damage indices, k_4^s being much lower than k_1^s, so that the denominator of the ratio defining d_4^s does not change much. On the other hand, the value of k_1^s is directly supplied by the proposed model.

A total number of elements $N = 100$ has been used in all analyses; different values of N have been employed (both greater and lower than 100) with negligible influence on the results. Table II shows the results of the identifications for both models and for each excitation.

It may be noted that the values of the identified parameters vary according to the excitation used to determine them, due to the different data sets accounted for in the identification. However, they recover the global $(k_{eff,M}, x_n)$ curves for the two systems considered in a fairly similar way. This is apparent from Figures 7 and 8, which refer to buildings A1 and C1, respectively. There, the three global curves obtained by considering the 3 excitation sets separately are reported with the experimental values of $k_{eff,M}$, derived from step 1. As can be seen, a good agreement holds between the estimated global evolutions and experimental results. Table III reports the estimated initial and final values of $k_{eff,M}$ (i.e. k_1^s, k_4^s) based on the considered events for the two buildings. The experimental values k_1^r, k_4^r (from step 1) are also reported and the errors of the estimates are shown.

It is seen from the table that, based on a single event, a good estimate of the initial virgin stiffness is obtained: for A1 the maximum error pertains to the first considered event (event 3) and is of the order of 8 per

Table III. Experimental and estimated initial and final stiffnesses and evaluation errors

A1 $k_1^r = 930\,(1/s^2)$ $k_4^r = 109\,(1/s^2)$					C1 $k_1^r = 1100\,(1/s^2)$ $k_4^r = 73{\cdot}2\,(1/s^2)$				
Test	k_1^s $(1/s^2)$	e_1	k_4^s $(1/s^2)$	e_4	Test	k_1^s $(1/s^2)$	e	k_4^s $(1/s^2)$	e_4
3	852	0·082	91	0·170	2	944	0·141	50·4	0·31
4	918	0·012	97	0·116	3	1020	0·073	48·2	0·34
5	892	0·040	93	0·153	4	1060	0·036	67·8	0·07

Table IV. Damage indices

A1	$k_1^r = 930\,(1/s^2)$ $k_4^r = 109\,(1/s^2)$					C1	$k_1^r = 1100\,(1/s^2)$ $k_4^r = 73{\cdot}2\,(1/s^2)$				
Test	k^D	d_1^r	d_1^s	d_4^r	d_4^s	Test	k^D	d_1^r	d_1^s	d_4^r	d_4^s
3	633	0·68	0·74	0·36	0·29	2	687	0·62	0·73	0·39	0·29
4	401	0·43	0·43	0·64	0·63	3	293	0·27	0·29	0·77	0·75
5	109	0·12	0·12	1·00	0·80	4	732	0·07	0·07	1·00	0·99

cent while for C1 it is about 14 per cent (event 2). In both cases estimates are closer to real values as the systems are more severely engaged in their non-linear behaviour. Estimates of the final stiffness are still satisfactory for system A1, with errors ranging from 11 to 17 per cent. Greater shifts are observed for C1 with errors about 30 per cent.

Table IV reports the calculated and experimental damage indices for the two buildings and for the intermediate events taken into account to determine them. The first index d_1^r (or d_1^s) describes how far the structure is from the original configuration after the considered shock, the second index d_4^r (or d_4^s) describes to what extent the total stiffness resources of the structure (described by k_1–k_4) have been exploited after the considered event; a unit value indicates that all stiffness resources have been exploited. Moreover, the comparison of d_1^r to d_1^s and of d_4^r to d_4^s provides a further assessment of the capacity of the method to recover the global evolution of $k_{\mathrm{eff,M}}$ from the response to a single intermediate event (which is reported in columns under the heading 'Test'). Initial and final experimental values of $k_{\mathrm{eff,M}}$ are also reported. As to the model A1 it can be seen that, after the first excitation engaging the system non-linearly (test 3), the structure has 68 per cent of its virgin stiffness properties (predicted value 74 per cent) while after the last excitation (test 5) this ratio drops to 12 per cent (predicted value 12 per cent). The other building C1 shows a quicker deterioration of initial properties. On the other hand, stiffness resources are exploited up to 36 per cent after test N.3 for A1 (predicted values 29 per cent) and are completely exploited during the last test, which attains experimental ultimate conditions (predicted value 80 per cent). Similar features hold for C1.

As can be seen the method provides results concerning both the previous and the future 'life' of the structure (respectively described by d_1^s and d_4^s) which are in satisfactory agreement with real data (described by d_1^r and d_4^r).

4. CONCLUSIONS

The method presented herein allows reasonable estimations both of the virgin and of the final effective stiffnesses of a significant mode when the base input and the response at a location of the structure are

known, provided the system is engaged, even slightly, in its non-linear phase of behaviour. The method is approximate, mainly due to the fact that relative modal displacements are determined by narrow-band filtering, being the effective participation factor of the considered mode unknown. It has been applied to almost all the 30 buildings tested within the experimental program described in Reference 4 and always gave good estimates of k_1 and k_4, provided the i.d.c. on which the procedure is based showed a decreasing path. Of these results, only the ones pertaining to two of the tested systems were presented in the previous section, to show the satisfactory match between predicted and experimentally determined initial and final quantities.

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REFERENCES

1. W. D. Iwan, 'On a class of models for the yielding behaviour of continuous and composite systems'. *J. appl. mech. ASME* **34**, 612–617 (1967).
2. C. Y. Peng, 'Generalized modal identification of linear and nonlinear dynamic systems', *Report No. EERL 87-05*, California Institute of Technology, Pasadena, CA, 1987.
3. W. D. Iwan and A. O. Cifuentes, 'A model for system identification of degrading structures, *Earthquake eng. struct. dyn.* **14**, 877–890 (1986).
4. D. Benedetti and P. Pezzoli, 'Shaking table tests on masonry buildings. Results and comments', *General report of ISMES and of the Structural Engineering Department, Politecnico di Milano, Italy*, 1996.
5. M. P. Limongelli, 'Identification of damage parameters from earthquake records', *Ph.D Thesis*, Department of Structural Engineering, Politecnico di Milano, Milan, 1995 *(in italian)*.